*Journal of Applied Mechanics and Technical Physics, Vol. 43, No. 1, pp. 90–92, 2002*

# SELF-SIMILARITY OF VIBRATIONAL MOTION IN A RESISTANT MEDIUM

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UDC 534

*Calculation results and an approximate description of the mean velocity of vibrational motion in a medium with drag proportional to velocity are presented as a function of various parameters characterizing the system.*

**Introduction.** Vibrational transport [1] is, undoubtedly, a special form of motion. The principle of excitation of this motion is unconventional. According to the main law of dynamics of a system of bodies, internal forces do not affect the position of the center of mass [2]. Nevertheless, they may affect external forces [3], which are friction forces [4–6] in the case of vibrational motion if a body moves over the surface or drag forces [7] if a mechanical system moves in a medium. The principle of vibrational displacement is based on this property. The papers cited do not form a complete list of theoretical results in this field. Vibrational motion of a system of bodies in a resistant medium is less adequately studied, and an attempt was made to remove this drawback only in [7]. It is of interest to study vibrational motion in a medium with drag proportional to velocity rather than to its square. In this case, however, it is impossible to solve the corresponding differential equations by analytical methods. Therefore, approximate solutions may turn useful, which allow one to evaluate comparatively easily the efficiency of vibrational motion with drag proportional to the velocity of the system of bodies relative to the medium.

**Diagram and Equation of Motion of a Vibromotive Engine.** A vibromotive engine is an asymmetric platform 1 of mass $M$ immersed partially or completely in a medium $L$ and a massive body 2 of mass $m$ that performs nondecaying, not necessarily harmonic oscillations in the reference system $X'O'Y'$ fitted to the platform (Fig. 1). It is assumed that the platform and the body do not move vertically. Therefore, it makes sense to consider only forces with horizontal components. These are the force $\boldsymbol{F}_{m-M}$ acting on the platform from the body, the force $\boldsymbol{F}_{M-m}$ acting on the body from the platform, and the drag force of the medium $\boldsymbol{F}_r$ with drag coefficients $\lambda_+$ and $\lambda_-$ corresponding to platform motion in the positive ($v > 0$) and negative ($v < 0$) directions of the $X$ axis:

$$\boldsymbol{F}_r = \begin{cases} -\lambda_+ v & \text{for } v > 0, \\ -\lambda_- v & \text{for } v < 0. \end{cases}$$

Let $\boldsymbol{x}$ be a vector determining the platform position relative to the medium-fixed reference system and $\boldsymbol{x}_m$ be a vector that describes the position of a body of mass $m$ relative to the platform. The equations of motion of the platform and the body are

$$M\,\frac{d^2\boldsymbol{x}}{dt^2} = \boldsymbol{F}_r + \boldsymbol{F}_{m-M}, \qquad m\,\frac{d^2(\boldsymbol{x} + \boldsymbol{x}_m)}{dt^2} = \boldsymbol{F}_{M-m}. \tag{1}$$

Taking into account the condition $\boldsymbol{F}_{m-M} = -\boldsymbol{F}_{M-m}$, we bring system (1) to the following form:

$$\frac{d^2}{dt^2}[M\boldsymbol{x} + m(\boldsymbol{x} + \boldsymbol{x}_m)] = \boldsymbol{F}_r. \tag{2}$$

Equation (2) is the law of motion of the center of mass of the system. Results of solving the differential inhomogeneous equation of motion (2) are described in the present work.

Rostov State University, Rostov-on-Don 344090. Translated from Prikladnaya Mekhanika i Tekhnicheskaya Fizika, Vol. 43, No. 1, pp. 108–111, January–February, 2002. Original article submitted March 30, 2001; revision submitted June 15, 2001.

 

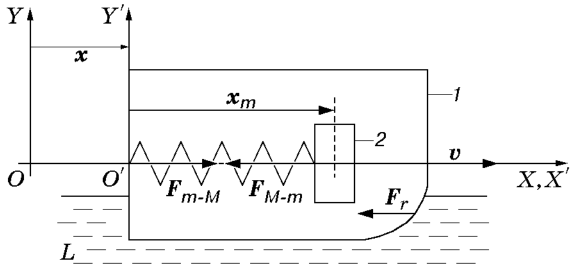


Fig. 1. Schematic of a vibromotive engine.

**Scaling of the Equation.** Let $M_0 = M + m$ be the total mass of the system. In the case of harmonic oscillations of the body of mass $m$, the dependence of the coordinate of this body relative to the platform on time is described by the law $x_m = a\cos(2\pi t/T)$. In this case, the use of new dimensionless variables

$$\zeta = \frac{M_0}{4\pi^2 ma}\,x, \qquad \tau = \frac{\lambda_+}{M_0}\,t, \qquad \vartheta = \frac{M_0^2}{4\pi^2 ma\lambda_+}\,\frac{dx}{dt}, \qquad \theta = \frac{\lambda_+}{M_0}\,T$$

allows us to reduce the number of quantities that determine the motion of the system. For $a > 0$, the equation of motion acquires the form

$$\frac{d^2\zeta}{d\tau^2} + \Big(\frac{1-\delta}{2}\,\operatorname{sign}\Big(\frac{d\zeta}{d\tau}\Big) + \frac{1+\delta}{2}\Big)\frac{d\zeta}{d\tau} - \frac{1}{\theta^2}\,\cos\frac{2\pi\tau}{\theta} = 0, \tag{3}$$

where $\delta = \lambda_-/\lambda_+$ is a parameter of asymmetry of the system; $\operatorname{sign}\vartheta = 1$ for $\vartheta > 0$ and $\operatorname{sign}\vartheta = -1$ for $\vartheta < 0$.

The dependences $\zeta(\tau)$ and $\vartheta(\tau)$ for $\delta = 5$ and $\theta = 1$ are plotted in Fig. 2. Nevertheless, the calculation results for the mean velocity $\langle v\rangle$ of vibrational motion for a fixed number of periods $n$ seems to be of greater interest:

$$\langle v\rangle = \frac{1}{nT}\int\limits_t^{t+nT} v(t)\,dt. \tag{4}$$

For high values of $t$, which are significantly greater than the motion-stabilization time, the mean velocity is independent of this parameter, which may be used in choosing the value of $t$ for numerical calculations of the mean velocity (4). The value of $n$ is responsible only for the accuracy of determining $\langle v\rangle$. In most cases, it is sufficient to have $n = 1$. The corresponding dimensionless reduced value of $\langle\vartheta\rangle$ depends only on the reduced period $\theta$ and the parameter of asymmetry $\delta$.

**Self-Similar Motion.** Without solving the differential equation (3), we can find an important property, namely: for very high ($\delta \gg 1$) and very low ($\delta \ll 1$) values of the parameter of asymmetry $\delta$, the products $\vartheta\theta^2$ and $\vartheta\delta\theta^2$ depend on $\tau$ and either on $\theta$ and $\delta\theta$, respectively. The mean velocity is no longer dependent on time, i.e., on $\tau$. High values of the parameter of asymmetry ($\delta \gg 1$) correspond to the motion in the positive direction of the $X$ axis only. In this case, we have $\operatorname{sign}(\vartheta\theta^2) = 1$, and the quantity $\langle\vartheta\rangle\theta^2$ depends only on $\theta$. For very small values of the parameter of asymmetry ($\delta \ll 1$), the motion of the platform in the positive direction of the $X$ axis becomes impossible, which corresponds to the condition $\operatorname{sign}(\vartheta\theta^2) = -1$. In this case, the value of $\langle\vartheta\rangle\theta^2\delta$ should depend only on $\delta\theta$. The same variant of motion can be described by the formal substitution of $1/\delta$ instead of $\delta$. It follows from here that, for high values of $\theta$, the parameter of asymmetry of the system $\delta$ should be present in the solution of the differential equation (3) only in the form of combinations $\delta-1$, $\delta+1$, $1-1/\delta$, and $1+1/\delta$. Simultaneous substitution of $\lambda_-$ for $\lambda_+$ and $\lambda_+$ for $\lambda_-$, other conditions being equal, in particular, for the same value of $T$, corresponds to the motion of the platform in the opposite direction with the same mean velocity $\langle v\rangle \to -\langle v\rangle$. The combination $\langle\vartheta\rangle\theta^2/(1-1/\delta)$ corresponds to the change in the velocity direction for $\lambda_- \leftrightarrow \lambda_+$ and the conditions given above. Similarly, the combination $\theta/(1+1/\delta)$ corresponds to a symmetric transformation of the oscillation period $T \leftrightarrow T$ for $\lambda_+ \leftrightarrow \lambda_-$. The approximate character of such an approach should be specially emphasized. First, this method is asymptotically accurate only for very high and very low values of the parameter of asymmetry. Second, very simple combinations of the quantities $\langle\vartheta\rangle$, $\theta$, and $\delta$ are written above, which, strictly speaking, testifies to the empirical approach to the solution of the problem. Third, the above arguments refer to the so-called transformations of symmetry, which can be accurate only for a rather limited range of problems [8].

Thus, an approximate self-similar dependence that describes the dependence of the mean velocity of vibrational motion on all parameters of the system should have the form $\langle\vartheta\rangle\theta^2/(1-1/\delta) = f(\theta/(1+1/\delta))$, where

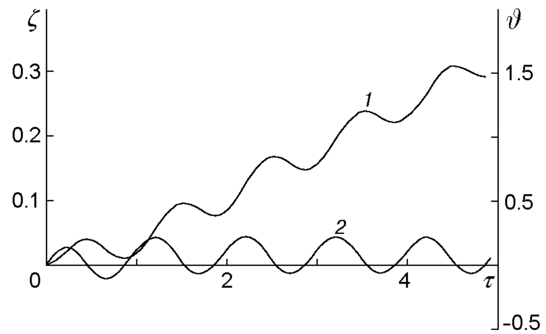


Fig. 2

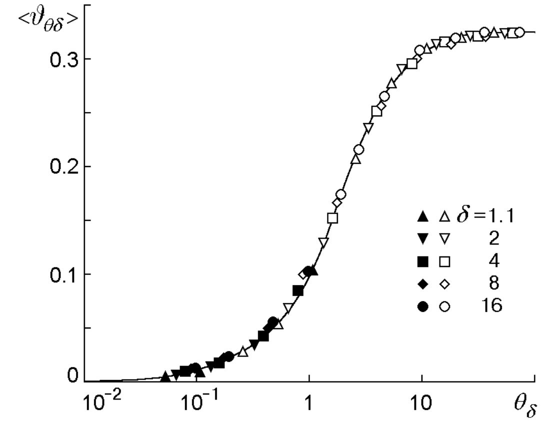


Fig. 3

Fig. 2. Dependence of the coordinate $\zeta$ (curve 1) and velocity $\vartheta$ (curve 2) on the time $\tau$ for $\delta = 5$ and $\theta = 1$.

Fig. 3. Self-similar dependence of the mean reduced velocity of vibrational motion $\langle\vartheta_{\theta\delta}\rangle$ on the reduced period of vibrations $\theta_\delta$ and the parameter $\delta$: the solid curve is dependence (5), the open and filled points show the solutions of Eq. (3) for $0.1 \leqslant \theta \leqslant 64$ and $0.1 \leqslant \theta < 1$, respectively.

$f$ is a certain function to be determined. For this purpose, it suffices to represent the solution of the differential equation (3) in the form of the dependence of $\langle\vartheta_{\theta\delta}\rangle = \langle\vartheta\rangle\theta^2/(1-1/\delta)$ on $\theta_\delta = \theta/(1+1/\delta)$. For the given values of $\delta$ and $\theta_\delta$, it allows one not only to find the value of the mean velocity but also to reconstruct the value of the reduced period $\theta$. This dependence is plotted in Fig. 3 for $0.1 \leqslant \theta \leqslant 64$ and $1.1 \leqslant \delta \leqslant 16$. The following variant of approximation of this dependence may be proposed, which corresponds to an asymptotic dependence of the mean velocity on the period of oscillations and parameter of asymmetry at high values of $\theta$:

$$\langle\vartheta\rangle = \frac{2\delta(\delta-1)}{\{(\delta+1)^3 + 64[\theta\delta(\delta+1)]^{3/2} + 31\theta^3\delta^3/2\}^{2/3}}. \tag{5}$$

Certainly, the problem may be solved in a different way, for example, by approximating the two-dimensional dependence of $\langle\vartheta\rangle$ on $\theta$ and $\delta$. Nevertheless, such an approach is valid only under certain restrictions on the parameters of the problem. The approach proposed yields rather accurate results, at least for all $\theta > 1$ and all values of the parameter of asymmetry $\delta$. For small values of the reduced period ($\theta < 1$), the principle of establishment of self-similarity is significantly different from that described here. Hence, for $\theta < 1$, it makes sense to use approximation (5) only for estimation and qualitative description of vibrational motion. Therefore, the results of the solution of the equation of motion (3) for $\theta > 0.1$ are also shown in Fig. 3.

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